

Modelling and forecasting fossil fuels, CO₂ and electricity prices and their volatilities

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HIGHLIGHTS

- ▶ A proposal of a joint modelling of fossil fuel, CO₂ and electricity prices is presented in this work.
 - ▶ Comparison of several univariate and multivariate models in terms of prediction accuracy for these series.
 - ▶ Conditionally heteroskedastic dynamic factor model for extracting common features in the multivariate volatility.
 - ▶ Common volatility factors extracted allows to enhance forecasting intervals and have an economical interpretation.
 - ▶ Starting point for risk management or portfolio optimization under uncertainty in the current context of energy markets.
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ABSTRACT

In the current uncertain context that affects both the world economy and the energy sector, with the rapid increase in the prices of oil and gas and the very unstable political situation that affects some of the largest raw materials' producers, there is a need for developing efficient and powerful quantitative tools that allow to model and forecast fossil fuel prices, CO₂ emission allowances prices as well as electricity prices. This will improve decision making for all the agents involved in energy issues.

Although there are papers focused on modelling fossil fuel prices, CO₂ prices and electricity prices, the literature is scarce on attempts to consider all of them together. This paper focuses on both building a multivariate model for the aforementioned prices and comparing its results with those of univariate ones, in terms of prediction accuracy (univariate and multivariate models are compared for a large span of days, all in the first 4 months in 2011) as well as extracting common features in the volatilities of the prices of all these relevant magnitudes. The common features in volatility are extracted by means of a conditionally heteroskedastic dynamic factor model which allows to solve the curse of dimensionality problem that commonly arises when estimating multivariate GARCH models. Additionally, the common volatility factors obtained are useful for improving the forecasting intervals and have a nice economical interpretation.

Besides, the results obtained and methodology proposed can be useful as a starting point for risk management or portfolio optimization under uncertainty in the current context of energy markets.

Keywords:
Time series models
Forecasting
Unobserved components
Fossil fuels
Electricity
CO₂ emission prices

1. Introduction

Nowadays, the uncertain context that affects the world economy and the energy sector, as well as the unstable political situation of some countries that are the most important producers of raw materials (such as oil or gas) makes even more necessary the development of quantitative tools and models that allow to improve investment decisions, adequately dealing with that increasing uncertainty.

The risks associated to the energy sector are mainly linked with the high volatility of fossil fuels, electricity and CO₂ emission

prices, that evolve over time and are difficult to forecast. Electricity is exchanged nowadays in competitive markets, as occurs with other *commodities*, but it presents some characteristics which make it different, such that for the moment it cannot be stored, or just a small amount, and demand needs to be covered immediately. These very peculiar features are responsible for its highly volatile behavior and the difficulty of price forecasting. Additionally, the fuel prices, which are also very volatile and difficult to forecast, can affect and influence electricity prices.

Furthermore, the increasing concern about climate change and the consequent regulation on Green House Gas (GHG) emissions has included a new affecting variable on the energy markets. The EU (European Union) emission trading scheme, started in 2005, sets caps for CO₂ emissions of plants across the EU-25. Installations are only allowed to increase their emissions above their caps subject to

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the acquisition of emission allowances. Thus, nowadays, different plant technologies do not only affect the prices by means of the fuel costs, but also via the effect of CO₂ prices. Moreover, CO₂ emission limits are also influencing the rapid development of renewable energies that is taking place, particularly wind, which has experimented a very high increase in the Spanish, Danish and German markets, among others. Comprehensive state-of-the-art reports on wind power forecasting are those from the ANEMOS.plus project [1] and from the Argonne National Laboratory [2].

Thus, climate policy is starting to have noticeable effects on energy markets, and the CO₂ price can be considered as a component of the generation cost for power generators and could be expected to have a deep impact on energy markets. The price of emissions should influence electricity prices, so as to reduce consumption and to promote cleaner technologies with benefits on air quality and climate change. In this direction, during the last years Spain (as well as in other European countries) has significantly increased the penetration of renewable sources of energy and even higher penetration is expected for the future. For instance, according to the EU Directive 2009/28/CE, at least 20% of the final energy consumption in 2020 must be generated by renewable energies. This statement is also in accordance with the larger social demand of sustainability and the need for reducing the high level of external energy dependence of our economy.

In this context, it is of interest to be able to jointly model CO₂ prices, fossil fuel prices (oil, gas and coal) and electricity prices as well as their volatilities, whose adequate modelling will allow to compute more accurate prediction intervals for the prices.

As far as fossil fuel prices modelling and forecasting are concerned, many authors have focused on developing adequate models. Pindyck [3] proposed an univariate model for the long-run evolution of oil, gas and coal prices by means of stochastic trends. A natural evolution of this model, already pointed out by the author, was its multivariate extension, since he did not account for it. A Bayesian estimation of this model, with application to the same raw materials' prices can be encountered in Radchenko [4], who again focused on the univariate approach.

Morana [5] introduced a semiparametric approach for forecasting oil prices, using GARCH (Generalized Autoregressive Conditionally Heteroskedastic) models and bootstrap techniques for the computation of prediction intervals.

Ye et al. [6,7] developed a simple time series model based for forecasting the monthly oil prices in the WTI (West Texas Intermediate) by incorporating the oil inventories in OECD (Organization for Economic Cooperation and Development countries) as an explanatory variable.

De Jong and Schenider [8] dealt with the relationships among different series of gas and electricity prices in several European markets. Their study is very interesting since it explores the possibility of a multi-market model.

Goshray and Johnson [9] studied univariate models for oil, gas and coal prices, detecting structural changes in the trends which suggests that structural changes in the economy affect commodity prices in the long term.

Asche et al. [10] and Bencivenga and Sargenti [11] investigated the short and long run relationships between oil, gas and electricity prices in England and the USA, respectively. In fact, Bencivenga and Sargenti [11], dealt with the obtention of short-term relationships from a quasi-static perspective and in their paper they emphasized the need for going a step forward to face the dynamic approach. The evidence of long-run relationships among the prices considered evidences the existence of a single common trend.

Lee and Lee [12] investigated the efficient market hypothesis using total energy price and various disaggregated energy prices (coal, oil, gas, and electricity) for OECD countries along the period 1978–2006, analyzing yearly data.

Another possible alternative are Multivariate Unobserved Component Models (MUCM), as the one developed by Den Blanken [13]. In this work, several univariate techniques are explored as well as a simple multivariate model, so as to take into account the interdependencies in fuel prices that could improve forecasts, that are crucial for being able to improve investment decisions.

Related to electricity price forecasting we can quote the papers by Conejo et al. [14], Nogales et al. [15] and Contreras et al. [16], who computed one-day-ahead forecasts in several European and North-American Markets. Diongue et al. [17] investigated conditional mean and conditional variance forecasts using a dynamic model following a k -factor GIGARCH process for electricity spot prices. Muñoz and Dickey [18] investigated the relationships between Spanish electricity spot prices and the US Dollar/Euro (USD/Euro) exchange rate during the period 2005–2007, taking into account the study of the association between dollar and oil prices. More recently, Alonso et al. [19] focused on the challenging task of computing year-ahead forecasts for the prices, and proposed a new Seasonal Dynamic Factor Analysis (SeaDFA) that is able to extract seasonal common factors that are used to produce these forecasts. Forecasting intervals are computed by applying a new bootstrap procedure introduced by the authors. Before the development of the SeaDFA, Dynamic Factor Models have been successfully applied in macroeconomics, finance and demography (see for instance the papers [20,21] or [22]), but scarcely to vectors of series of interest in the current context of power markets.

As far as CO₂ prices are concerned, the models proposed for this issue are relatively new, as the topic is. Some recent references are Feng et al. [23] who examined carbon price volatility using data from the European Union Emission Trading Scheme from a non-linear dynamic point of view. Zhang and Wei [24] summarized the main arguments of empirical studies on the EU ETS (EU Emissions Trading System), in terms of two aspects: the operating mechanism and economic effect of the EU ETS. Tolis and Rentizelas [25] carried out a systematic investigation of power sector portfolios through discrete scenarios of electricity and CO₂ allowance prices. Benz and Truck [26] analyzed the short-term spot price behavior of carbon dioxide (CO₂) emission allowances of the new EU-wide CO₂ emissions trading system (EU ETS) by means of Markov switching and univariate AR-GARCH models for stochastic modeling.

Nevertheless, to the best of our knowledge, and to date there is no a published paper that deals with the joint modelling and forecasting of all these prices and their volatilities that are so relevant in the current context of power markets (electricity, fossil fuels and CO₂ emissions prices). Just some subsets of these series have been considered in the previous papers briefly summarized above, but not all of them together.

In this work we try to fill this gap, studying the evolution over time not only of the conditional mean but also of the conditional variance of all these prices. For this purpose we use a conditionally heteroskedastic dynamic factor model that accounts for the multivariate structure in the volatilities once the structure in the conditional mean has been removed. We consider the evolution over time of the CO₂ prices as well as oil, gas and coal prices and daily electricity prices in the Spanish Market.

The prices related to CO₂ market included in this work are the EUAs and CERs. The EUA prices are the materialization of the European Union Allowances and one EUA represents the right to emit one ton of CO₂. Besides, Emissions Trading allows to transfer an Assigned Amount Unit (AAU) across international borders or emission allowances between companies covered by a Cap and Trade scheme. CERs are issued for emission reductions from Clean Development Mechanism (CDM) project activities. This CDM was included in Article 12 of the Kyoto Protocol, designed to assist

developing countries in achieving sustainable development by permitting industrialized countries to finance projects for reducing GHG emission in developing countries and receive credit for doing so. Thus, CERs are issued for emission reductions from Clean Development Mechanism (CDM) project activities.

In order to model the structure in the conditional mean we provide a comparison between the univariate and multivariate approaches for the series of interest here, and the main objective is to check which approach (univariate or multivariate) is more convenient in terms of forecasting accuracy. The use of multivariate time series models as well as the development of detailed comparisons between the univariate and multivariate approaches is extended in applications to macroeconomics or finance [27,28,22], but not in series of interest in the energy sector.

Additionally, we focus on the extraction of common unobserved components (factors) in volatility by presenting an extension of the methodology developed by Harvey et al. [29]. It can be detected empirically that there is a multivariate structure affecting the conditional variance, and handling this effect by using multivariate GARCH models it is not convenient from the practical implementation perspective (difficulties in estimation arise due to the high number of constraints to impose as well as the large number of parameters to estimate, the so called *curse of dimensionality*). That is why the extraction of common volatility factors is presented in this paper as an alternative that allows to take into account the multivariate structure in volatility but avoiding multivariate conditionally heteroskedastic models (m-GARCH models). The factor model here used allows to capture comovements in volatility and then these common factors are used to improve both the estimation of the volatility of each series and the computation of forecasting intervals. However, other alternatives such as (1) assuming homoskedasticity just fitting a model for the conditional mean or (2) not accounting for the multivariate structure in volatility by fitting univariate GARCH models to each series do not allow to deal with all the important features of these series.

Although it is out of the scope of this paper additional interesting applications or extensions of the results obtained in this paper could be related to its use for portfolio optimization in the energy sector [18,30,25] or related to risk management in replacement investment decisions in the electricity sector (this subject has been recently studied by Fuss and Szolgayová [31]).

The rest of the paper is organized as follows. In Section 2 we describe the real data under study (series of EUA and CER prices, oil, gas, coal and electricity prices) and provide a brief descriptive study on them that allows to justify the methodology applied in the following sections. In Section 3 we focus on the methodology used for modelling and forecasting these prices, and both the univariate and multivariate time series approaches are described and applied, providing also a comparative study between their estimation and forecasting performance. Section 4 is concerned with the extraction of common features in the volatility of the aforementioned power markets and its relationship with the accurate estimation of the width of forecasting intervals of the series of prices under study in this paper. The model proposed by Harvey et al. [29] is explained and an extension of it is applied to the data under study, linking this issue with the computation of forecasting intervals that should be wider in high volatility periods but narrower in low volatility ones. Section 5 concludes and present some interesting subjects for further research.

2. Data and descriptive statistics

In this paper we consider six series of prices of great importance in power markets that additionally are relevant in macroeconom-

ics. The data are those in the period from the 6th of March 2009 till the 29th of April 2011. The series under study are the following¹:

- EUA prices, which are materialization of the EU ETS quotas, the tradable unit under the EU ETS.
- CER prices previously described.
- Fossil fuel prices. In this paper we consider the oil, gas and coal prices. SENDECO2 (Sistema Electrónico de Negociación de Derechos de Emisión de CO₂, Electronic System of CO₂ Emission Allowances Rights) considers these three fuel prices as reference magnitudes for the EUAs and CERs, which means that they could be relevant for the first two series mentioned.
- Electricity prices in the Spanish Market. From the end of the 90's the electricity markets are liberalized in most of the developed countries. Prices are cleared for every hour or every half an hour in some specific markets, using the bids submitted by producers/generators as well as consumers/sellers. In the Spanish Market the marginal price is defined as the one bided by the last generator whose production is needed to satisfy the whole demand. Here we use daily prices, calculated as the mean of the 24 hourly prices as the indicator of interest.

All these series of prices are shown in Fig. 1, from which we detect that all of them are non-stationary in mean, given that the conditional mean evolves over time, not being constant. Non-stationarity can be also confirmed by plotting the respective autocorrelation functions of the series or applying the Dickey-Fuller test [32] to the log-prices.² Apart from the need for differencing the log-prices and removing non-stationarity, the remaining (auto and cross) correlation must be captured.

Fig. 2 shows the conditional variances of the centered returns of the series of prices under study. The common patterns affecting them are clustered into 3 groups. On the one hand, conditional variances of CO₂ prices (EUAs and CERs) are really similar, with coincident periods of high and low volatilities. On the other hand, the patterns of the evolution over time of the volatility of fuel prices are also very similar with a lot of common features. Finally, the volatility associated with electricity prices combines common characteristics with the other variables whose prices are under study in this paper. For example, some periods of high and low volatilities are coincident with those of CO₂ and fuel prices, the two clusters previously described, and electricity price volatility also presents some specific features. It worth explicitly mentioning that the presence of common features is more evident when focusing on volatilities, as shown in Fig. 2, than when focusing on the characteristics of the conditional mean of the series under study (Fig. 1). This will also be noticeable in the results presented in Sections 3 and 4.

3. Methodology for modelling and forecasting prices

In this Section we summarize the main features of univariate and multivariate time series models as well as its application to the prices under study: fuels (oil, gas and coal), CO₂ (EUAs and CERs) and electricity prices in the Spanish Market. Excellent references on univariate and multivariate time series models are Peña et al. [33], Tsay [34] and Shumway and Stoffer [35].

¹ All the prices considered are in €. EUAs, CERs and daily prices from the Spanish Market were originally in €. The fuel prices were in \$ and the corresponding daily €/€ exchange rate was used for the conversion. The EUAs, CERs and fossil fuel prices are available for weekdays from www.sendeco2.com and the electricity prices are available from the Spanish Market Operator website www.ree.es.

² The series of prices are usually transformed using the logarithmic transformation to remove the most evident type of heteroskedasticity. In Section 4 of this paper we also model the evolution over time of the common features in the volatilities of the series under study.

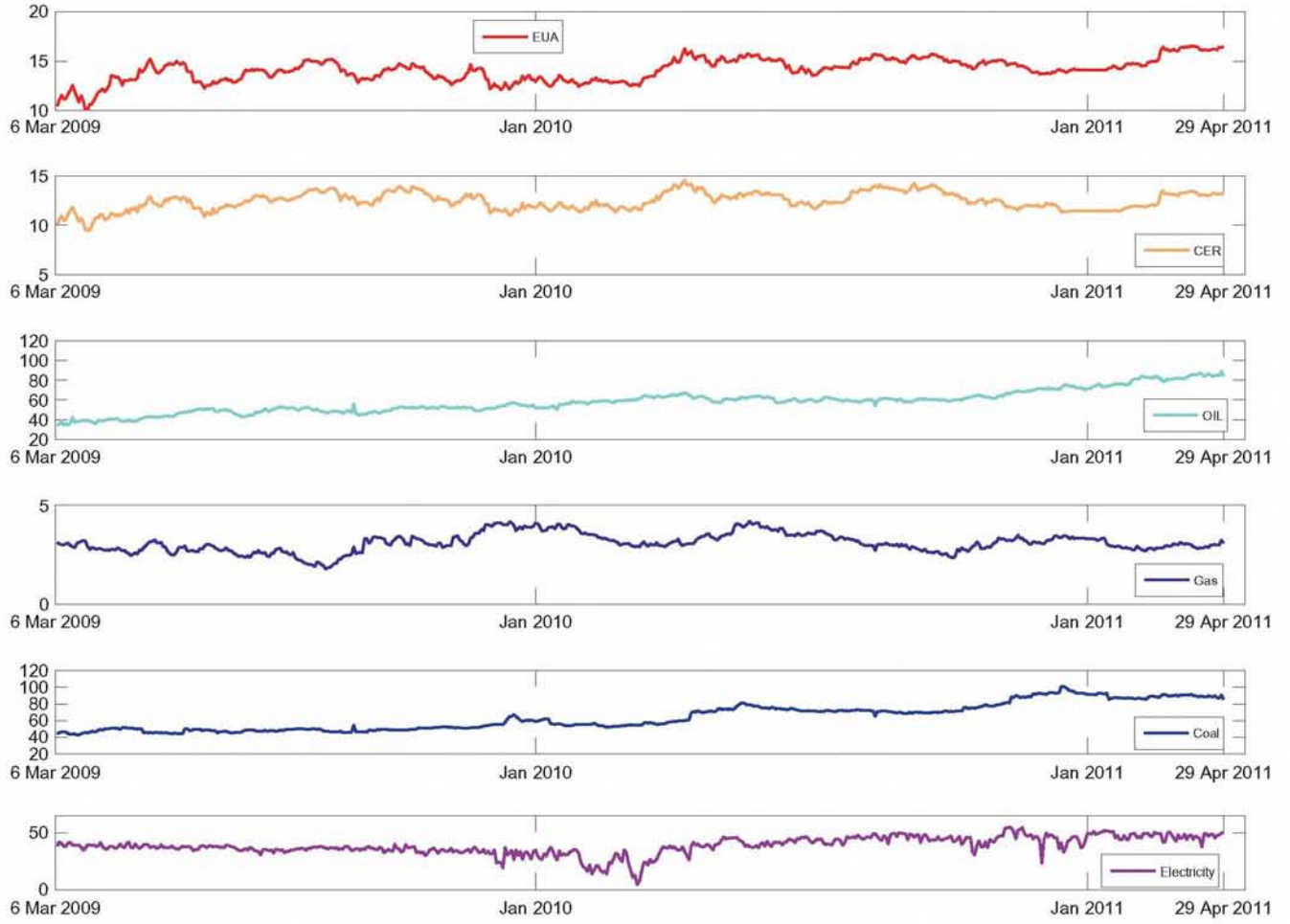


Fig. 1. Evolution over time of prices: CO₂ (EUAs and CERs), fuels (oil, gas and coal) and electricity, in the period 6th March 2009 till the 29th of April 2011.

3.1. Univariate time series models

Auto Regressive Integrated Moving Average (ARIMA) models processes are a class of stochastic processes used to model and forecast time series. The application of the ARIMA methodology for the study of time series analysis is due to Box and Jenkins [36].

Let $p_{j,t}$ be the price of the j -th series under study. Usually, time series of prices are log-transformed to remove the most evident type of heteroskedasticity and then, once the variance has been stabilized, a regular difference is applied to the log-prices in order to get stationarity in mean. By doing so, the observed series becomes stationary both in variance and mean. Let $y_{j,t} = \log(p_{j,t}) - \log(p_{j,t-1})$ be the return series of the prices $p_{j,t}$. Then, the dependence structure of the stationary series in variance and mean, $y_{j,t}$ can be modelled by an ARMA(p, q) model, whose general expression is given by:

$$\phi(B)\tilde{y}_{j,t} = \theta(B)a_t, \quad a_t \sim NIID(0, \sigma_a^2),$$

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)\tilde{y}_{j,t} = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)a_t, \quad (1)$$

where $\tilde{y}_{j,t} = y_{j,t} - \bar{y}_{j,t}$ and B is the lag operator such that $By_{j,t} = y_{j,t-1}$. The error term a_t is assumed to be Normally, Independent and Identically Distributed (NIID).

The model described by Eq. (1) for $\tilde{y}_{j,t}$ can be written as follows for the original series of prices $y_{j,t}$:

$$\phi(B)\nabla(\log(p_{j,t})) = \theta(B)a_t, \quad a_t \sim NIID(0, \sigma_a^2), \quad (2)$$

where the difference operator $\nabla = 1 - B$. If $y_{j,t}$ is generated by and ARMA(p, q) model and only one difference was needed to stabilize

the mean, then the $\log(p_{j,t})$ is generated by an ARIMA($p, d = 1, q$) model, where d is the order of integration.

As for the identification of the orders of an ARMA model, p and q , for $\tilde{y}_{j,t}$, two very useful tools are the simple and partial autocorrelation functions (ACF and PACF respectively). The ACF and PACF of the ARMA processes are the result of combining AR and MA properties, which can be also seen as particular cases of an ARMA(p, q): ARMA($p, 0$) and ARMA($0, q$) respectively.

Thus, the steps that must be followed to estimate an ARIMA model that will be used to forecast are the following:

1. Check the stationarity of the variance, to decide on applying the logarithmic transformation.
2. If the mean is not constant over time a difference (or maybe in some particular cases even 2) should be applied.
3. Once the data are stationary after performing Steps 1 and 2 when needed, the appropriate orders of the ARMA (p, q) model must be selected based on the different patterns that the ACF and PACF present in AR, MA or ARMA models.
4. Then the parameters involved in the selected model are estimated by Maximum Likelihood (ML).³
5. Once the model has been estimated then the hypotheses assumed for the error term a_t must be checked in the diagnostic checking stage. This can be done by applying, for instance, the

³ The estimation can be carried out using different available softwares (MatLab, EViews, TRAMO, among many others). Details on the estimation procedure and likelihood maximization can be encountered in any of the references on Time Series Analysis listed above.

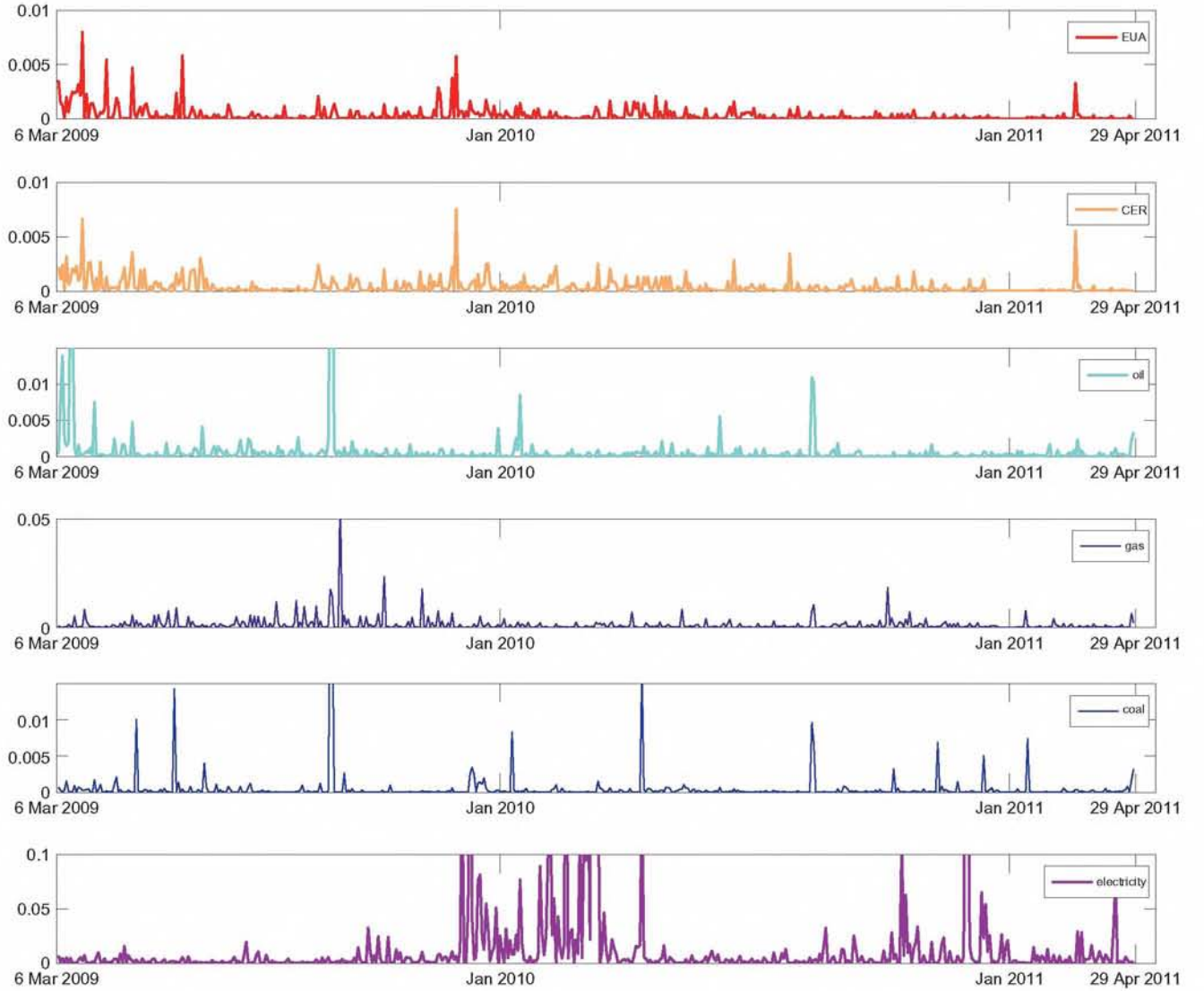


Fig. 2. Evolution over time of the conditional variances of the returns ($r_t = \log p_t - \log p_{t-1}$): CO₂ (EUAs and CERs), fuels (oil, gas and coal) and electricity, in the period 6th March 2009 till the 29th of April 2011.

Ljung–Box test [37] to check the independence assumption and the Kolmogorov–Smirnov test for testing the normality assumption. If the independence and normality assumptions are not rejected then the estimated model can be used to compute forecasts for the price $p_{j,t}$. Otherwise, an alternative model should be estimated, going back to Steps 3, 4 and 5, subsequently.

3.2. Multivariate time series models

VARMA (Vector ARMA) models are the multivariate extension of ARMA, and can be considered as their generalization to deal with vectors of series instead of a single one. The extension is, however, not straightforward, and some identifiability issues should deserve some attention. In the literature some different structural specifications have been proposed to solve this problem (see [38] or [39]). Nevertheless, the need to discuss these issues in detail here does not exist because VAR and VMA models are sufficient in a great number of applications.⁴ In some cases, when a

VARMA model is needed, only low order ones are of application, mainly in the case in which there is no a seasonal pattern in the data.

In general a VARMA(p, q) model can be written as follows:

$$\Phi(B)y_t = c + \Theta(B)u_t, \quad (3)$$

where $\Phi(B) = I - \Phi_1 B - \dots - \Phi_p B^p$ and $\Theta(B) = I - \Theta_1 B - \dots - \Theta_q B^q$ are m by m matrix polynomials, where m is the dimension of the original vector of series. The stationarity condition is that the roots of $|\Phi(B)|$ are outside the unit circle, which is the same as checking that the eigenvalues of the transition matrix, when writing the model under its state-space formulation, are in modulus smaller than 1. The roots of $|\Theta(B)|$ should also be outside the unit circle. $c = (c_1, \dots, c_m)'$ is a vector that contains the m constants in the model of each series and $u_t \sim N_m(0, \Sigma_u)$.

The particular equations for a VAR(1) model in the bivariate case are as follows:

$$y_{1,t} = c_1 + \phi_{11}y_{1,t-1} + \phi_{12}y_{2,t-1} + u_{1,t}, \quad (4)$$

$$y_{2,t} = c_2 + \phi_{21}y_{1,t-1} + \phi_{22}y_{2,t-1} + u_{2,t}, \quad (5)$$

and based on this: ϕ_{12} measures the linear dependence of $y_{1,t}$ on $y_{2,t-1}$. For example, if $\phi_{12} = 0$ then $y_{1,t}$ does not depend on $y_{2,t-1}$. If

⁴ This is the case of the vector of series under study, for which we have fitted a vector model just with MA component.

$\phi_{21} = 0$ then $y_{2,t}$ does not depend on $y_{1,t-1}$. If both $\phi_{12} = \phi_{21} = 0$ then $y_{1,t}$ and $y_{2,t}$ are uncoupled and could be modelled by univariate AR models.

The equations of an VMA(1) model in the bivariate case are the following:

$$y_{1,t} = c_1 + \theta_{11}u_{1,t-1} + \theta_{12}u_{2,t-1} + u_{1,t}, \quad (6)$$

$$y_{2,t} = c_2 + \theta_{21}u_{1,t-1} + \theta_{22}u_{2,t-1} + u_{2,t}, \quad (7)$$

which means that the current observations of the series, $\mathbf{y}_t = (y_{1,t}, y_{2,t})$ only depend on the current and past shocks. If $\theta_{12} = \theta_{21} = 0$ then the series are uncoupled. There is an unidirectional dynamic relationship from $y_{1,t}$ to $y_{2,t}$ if $\theta_{12} = 0$ and $\theta_{21} \neq 0$, and the opposite holds when $\theta_{21} = 0$ and $\theta_{12} \neq 0$. There is a feedback relationship between $y_{1,t}$ and $y_{2,t}$ if both θ_{12} and θ_{21} are different from 0.

To identify the order of a VARMA model for an stationary vector of series \mathbf{y}_t the sample cross-correlation matrices (CCM) are used. Once the selected model has been estimated based on this matrices a generalization of the Ljung–Box test for the multivariate case must be used to check that there are no auto and cross-correlations in the vector \mathbf{u}_t . As it occurs in the univariate case, a model selection criteria such as the AIC or BIC can be applied to decide on the orders of the model to be fitted.

As occurs with dynamic factor models, the use of VARMA models is very common in macroeconomic, demographic or financial applications but scarce in applications focused on series of interest in the context of power markets. One possible reason for this could be the high dimension of the vectors of series. That is why Alonso et al. [19] and García-Martos et al. [40] have recently proposed two factor models that reduce the original observed m -dimensional series ($m = 24$) to a set of $r = 2$ unobserved common factors that evolve over time according to a seasonal VARIMA model with homoskedastic or conditionally heteroskedastic disturbances, respectively.

3.3. Application to fossil fuel, CO₂ and electricity prices

The data under study comprises the period from the 6th of March 2009 till the 29th of April 2011. The data from the first two years (2009 and 2010) is used to estimate the corresponding univariate ARIMA models and multivariate VARIMA models. The first four months in 2011 are used to check the performance of the two models under comparison, both the univariate and multivariate approaches, by computing out-of-sample forecasts. The forecasts will be computed for a forecasting horizon (h) ranging from 1 up to 15 days. This means forecasting 3 weeks ahead, given that the data available corresponds to working days, in which all the markets considered are open.

We have computed forecasts for every weekday (workday) in the period from the beginning of January 2011 till the end of April 2011, for every forecasting horizon from $h = 1, \dots, 15$. This is done by using a rolling-window, that allows both to update the input data and to use these updated data to select daily the model and to re-estimate its parameters.

For instance, with the data from March, 6, 2009 till the end of December 2010, we compute a forecast for the first day in January 2011 (thus $h = 1$ in this case), and for the second workday in January (for which the forecasting horizon h would be 2) and for the 15th workday in this month ($h = 15$).

Then, the input data is updated incorporating the real prices for the first working day in January. Not only the data is refreshed, but also the selection of the model using the BIC and its parameter estimation is carried out daily, when a new data is available. Thus, the models are also reestimated using these updated input data. The first update of the data would imply to include the first day in January 2011, and then selecting again the model and reestimating

and computing a forecast for the second workday in this month (for which $h = 1$), for the third one (for which $h = 2$), ..., and for the 16th weekday in January (for which the forecasting horizon would be $h = 15$). This process is repeated, updating the data and selecting and re-estimating an univariate model for each series every day, as well as a multivariate model for all of them, and computing the forecasting error, e_{dj} , for series j and day d as follows:

$$e_{dj} = \frac{|p_{dj} - \hat{p}_{dj}|}{p_{dj}},$$

where p_{dj} is the real price of each magnitude, $j = 1, \dots, 6$ and \hat{p}_{dj} the corresponding forecast. The MAPE (Mean Absolute Percentage Error) for a period of D days is calculated as the mean of the e_{dj} , $d = 1, \dots, D$.

Besides, for each day and for each forecasting horizon considered, h , we will have 2 forecasting errors corresponding to the univariate and multivariate models.

This procedure allows us to obtain forecasts and daily forecasting errors for each series of interest and for a large period, allowing us to compare the univariate and multivariate approaches in terms of prediction accuracy for different prediction horizons: from one-day-ahead, $h = 1$, till 3-weeks ahead, $h = 15$. The objective is to be able to select which of the two approaches considered is better in terms of forecasting accuracy for each variable under study. The existence of cross-correlations in the vector of series considered does not necessarily mean that the multivariate model, although fitted, is always better in terms of prediction accuracy.

For the selection of the most adequate univariate model, and given the great number of models to identify and estimate, 480 ($= 6$ series \times 80 days, because the models are reestimated when updating the data, i.e., daily, as explained above), the identification cannot be done using the corresponding manual fitting of all of them using ACF and PACF, and requires the automatization of the procedure.

In this paper we have used TRAMO ("Time series Regression with Arima noise, Missing observations and Outliers"), a free software developed by Caporello, G. and Maravall, A. (Bank of Spain) for the estimation and subsequent forecasting with ARIMA models. Identification and intervention of outliers, as well as estimation of models can be done with an automatic procedure. The models are selected using the Bayesian Information Criteria (BIC). This criteria takes into account both the likelihood of the model (by means of its residual variance) and the parsimony of the model by including a term that penalizes models with a great number of parameters.

The expression of the Bayesian Information Criteria (BIC) is:

$$BIC = n \log(\hat{\sigma}_R^2) + k \log(n) \quad (8)$$

where n is the length of the time series used to estimate the model, $\hat{\sigma}_R^2$ is the residual variance, and k is the number of parameters estimated. Models with lower BIC are preferred.

As far as the multivariate model is concerned, we have fitted a VARIMA (0,1,1), which is able to capture stationarity and the cross-correlation structure of the vector of series under study. This model has been re-estimated every time a new real data was available, so although the same model is considered for the whole period under study, the estimates of the parameters could change over time when updating the data during these four months. The relationship between AR and MA process implies that a VMA(1) can be expressed as an VAR model of infinite order. The estimation of multivariate models has been carried out using the freely available E4 toolbox [41] for MatLab.

Once all the models have been estimated and forecasts computed as described, we will now compare the forecasting errors of the univariate and multivariate approaches. Tables 1 and 2 include the daily MAPEs (in %) for the univariate and multivariate

Table 1

UNIVARIATE MODELS. Out-of-sample forecasting errors (MAPE in percentage), January 1, 2011–April 29, 2011. Forecasting horizons from 1 up to 15 days.

Forec. horizon	EUA	CER	Brent	Gas	Coal	Electricity
$h = 1$	0.6628	0.6346	1.4267	1.6851	1.2363	6.0788
$h = 2$	0.9549	0.9917	1.8366	2.3094	1.6076	7.8253
$h = 3$	1.1970	1.2816	2.0978	2.6387	1.8436	8.1799
$h = 4$	1.4525	1.5739	2.4819	2.9287	2.1636	7.8261
$h = 5$	1.6132	1.8159	2.7956	3.4342	2.3060	7.4850
$h = 6$	1.7786	2.0146	2.8650	3.9527	2.3738	6.8186
$h = 7$	1.9974	2.1544	3.0986	4.5551	2.4918	6.4944
$h = 8$	2.1799	2.2964	3.3138	4.8331	2.5789	6.2018
$h = 9$	2.3676	2.5198	3.3976	5.1780	2.6218	6.5749
$h = 10$	2.4999	2.6894	3.5955	5.4502	2.8034	7.2665
$h = 11$	2.6682	2.8753	3.7525	5.7865	2.9713	7.7270
$h = 12$	2.8394	3.0769	3.8283	6.1114	3.1193	7.8683
$h = 13$	3.0672	3.3116	4.0781	6.3551	3.3198	8.2262
$h = 14$	3.2493	3.5327	4.1001	6.6207	3.5512	7.8652
$h = 15$	3.4782	3.7987	4.1144	6.8299	3.7141	7.1638
Average	2.1338	2.3045	3.1188	4.5779	2.5802	7.3068

Table 2

MULTIVARIATE MODELS. Out-of-sample forecasting errors (MAPE in percentage), January 1, 2011–April 29, 2011. Forecasting horizons from 1 up to 15 days.

Forec. horizon	EUA	CER	Brent	Gas	Coal	Electricity
$h = 1$	0.7051	0.6455	1.4123	1.6972	1.0439	5.9933
$h = 2$	0.9665	1.0312	1.8286	2.2980	1.4474	7.9754
$h = 3$	1.1974	1.2984	2.0888	2.6515	1.7354	8.2540
$h = 4$	1.4747	1.5861	2.4528	2.9127	2.1167	9.1473
$h = 5$	1.6462	1.8576	2.7687	3.4276	2.2229	8.6730
$h = 6$	1.8256	2.0402	2.8664	3.9427	2.3018	8.1531
$h = 7$	2.0301	2.2099	3.0774	4.5460	2.4292	8.1606
$h = 8$	2.2091	2.3472	3.3108	4.8317	2.5145	8.2371
$h = 9$	2.3934	2.5193	3.3854	5.1765	2.5755	8.6804
$h = 10$	2.5248	2.7095	3.5857	5.4534	2.7049	8.3660
$h = 11$	2.6862	2.8787	3.7385	5.7853	2.8590	8.8171
$h = 12$	2.8442	3.0662	3.7877	6.1114	3.0372	9.6011
$h = 13$	3.0706	3.2953	4.0368	6.3408	3.2224	10.3328
$h = 14$	3.2490	3.5218	4.0743	6.6157	3.4522	10.4835
$h = 15$	3.4851	3.7895	4.0813	6.8079	3.6266	10.1177
Average	2.1539	2.3198	3.0997	4.5732	2.4860	8.7328

specifications above described. For the CO₂ and electricity prices, the univariate approach produces better results in terms of prediction accuracy for every forecasting horizon, except for one-day-ahead forecasts of electricity prices ($h = 1$). For oil, coal and gas prices, the multivariate approach enhances the results given by the univariate model.

Additionally, it would be interesting to know if including the wind power production in the VARIMA model⁵ allows to reduce the forecasting errors previously obtained. For $h = 1$ the best model among the two previously considered was the VARIMA model that till this point does not include the wind power production, and for $h > 1$ the best model in terms of out-of-sample forecasting error was the univariate model. Wind power production data has been found very relevant for forecasting electricity prices Jonsson [44] and this could be important for the Spanish Market specially for one-day-ahead forecasts, given that when extending a bit the fore-

casting horizon the uncertainty affecting the forecasts of wind power production dramatically increases.

Estimating a VARIMA model that includes the wind power, as previously described, reduces the forecasting error of the best model considered till this point for $h = 1$, i.e., one-day-ahead forecasts. For longer forecasting horizons ($h = 2, \dots, 14$) the results obtained with the VARIMA including the wind power production beat those of the VARIMA without wind but not the best ones given by the univariate model. For the longest forecasting horizon ($h = 15$) the results from the VARIMA including the wind are even worse than those from the VARIMA without the wind, and this is sensible given the huge uncertainty associated to the computation of wind power production forecasts for longer horizons.

The forecasting errors obtained for electricity prices when incorporating in the estimation of the VARIMA model the wind power production are shown in Table 3.

The inclusion of the wind power production data worths the effort for the computation of one-day-ahead forecasts ($h = 1$) since the differences between the two VARIMA models described are statistically significant, thus the forecasting error including the wind power is significantly smaller. This has been tested using the methodology proposed in García-Martos et al. [45] based on Analysis of Variance (ANOVA, see Montgomery, 1984). The comparison is carried out for electricity prices for the two VARIMA models given that for $h = 1$ (day-ahead forecasts) the multivariate model was the best one among the 2 considered before. For the rest of the series of prices considered in this study the VARIMA model including the wind generation does not beat the best one in each case (univariate for emmission prices and multivariate for the fuel prices).

Related to the inclusion of wind power production, a possible alternative to the multivariate modelling and forecasting through VARIMA models, and just in the case of one-day-ahead forecasts ($h = 1$) would be to include this variable as an exogenous one for the computation of electricity price forecasts. This would avoid to model and forecast it, just replacing this by using directly the wind power forecasts provided by the System Operator of the Iberian Peninsula and obtained from their wind power forecasting tool SIPREOLICO, Sánchez [46,42]. These day-ahead ($h = 1$) forecasts are freely available in: <http://www.esios.ree.es/>.

Finally, to summarize the results for all the variables considered here for both approaches (univariate and multivariate) the results can be considered accurate, particularly those obtained for electricity prices are of similar magnitude of those obtained in recent or well known references for the short-run [14] or [45].

The superiority of the univariate models in some cases could be explained by the fact that maybe for these prices the past values of the other commodities prices do not provide relevant information in terms of forecasting accuracy. But for instance, if considering much longer forecasting horizons the electricity prices could incorporate the effect of high or low fossil fuel prices.

The improvement obtained for the Brent, coal and gas prices points to the direction of the existence of cross-correlations among these power markets. Given the importance of these series, that are not just relevant in the context of energy markets, but also very important in the economy of any country, the multivariate ap-

Table 3

MULTIVARIATE MODELS including wind generation. Out-of-sample forecasting errors (MAPE in percentage), January 1, 2011–April 29, 2011. Forecasting horizons from 1 up to 15 days.

$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
5.9600	7.9567	8.2549	9.1282	8.6551	8.1259	8.1236	8.2063
$h = 9$	$h = 10$	$h = 11$	$h = 12$	$h = 13$	$h = 14$	$h = 15$	
8.6564	8.3535	8.8213	9.5923	10.3145	10.4713	10.1258	

⁵ The objective of building this VARIMA model is not to produce accurate forecasts of the wind power production given the non-linearity that it presents and that sophisticated methodologies has been specifically proposed to compute wind power production forecasts (Refs. [42,43], and comprehensive reports on this issue containing a state of the art in this field are those from the ANEMOS.plus project [1] as well as the report from the Argonne National Laboratory [2]). The objective here is to check if the VARIMA model built considering 7 series (the 6 ones aforementioned as well as the wind power production) gets better results for electricity prices or for any of the other series considered.

proach provides a generalization of the univariate one that allows to improve forecasting results.

Figs. 1 and 2 show respectively the evolution over time of the prices (conditional mean) and volatilities (conditional variance). The presence of common features (co-movements) is more evident in the volatilities. Thus, accounting for the multivariate structure in variance could be more relevant for the volatilities, for which the improvements with respect to an alternative univariate approach should be more remarkable. The next Section is concerned with this issue.

4. Modelling volatilities by extracting common features

4.1. Motivation and objectives

Once the structure in the conditional mean has been captured, which allows to produce price forecasts for the series of interest (these results are provided in Section 3.3) we focus on modelling the structure in the conditional variance of the series of prices under study. This will be done by focusing on the series of residuals of the models estimated in the previous section. There are no important changes in terms of the estimates obtained for the common volatility factors depending on considering the residuals of the VARMA model or when considering those from the univariate ARMA models, however, we will present here the results corresponding to the estimation of common volatility factors obtained from the residuals of the multivariate model fitted for modelling the structure in the conditional mean.

As described in the Introduction, where we cited some references on this issue, previous works dealt with modelling the structure in the volatility of the time series of CO₂, fuels and electricity prices. However, most of them (as occurred with the evolution over time of the conditional mean) were univariate models that did not take into account the multivariate structure affecting the conditional variance of all these data that was detected in Fig. 2. Multivariate GARCH models are an alternative to univariate modelling of the conditional variance. However, its implementation is not easy and its estimation present some drawbacks that will be briefly described in the forthcoming subsection.

Here we provide an application of the model developed by Harvey et al. [29], although extended to the case in which several common factors are extracted instead of a single one, as an alternative that is able to take into account the multivariate structure of the volatility by extracting common volatility factors underlying the six series of interest in this paper, but avoiding the problems that multivariate GARCH models present. Additionally, the extraction of common components is an interesting issue that allows to decompose the time-varying variance-covariance matrix into its common and specific components.

Once the common volatility factors have been estimated they will be used to estimate the conditional variances of the original data. The extraction of common factors will then allow to properly estimate the evolution over time of the conditional variance of each series accounting for the multivariate structure in variance.

The extraction of common volatility factors will allow to calculate more accurate forecasting intervals for the series under study as a direct output of the Kalman Filter, as it will be described.

4.2. Methodology

Financial time series are well-known for presenting some empirical features such as high-frequency (daily or even smaller) as well as non-gaussian distributions with heavy tails (although symmetric).

The main difference between financial time series and others such as the ones we are studying here are related to the fact that in our case the structure in the conditional mean is more complex and cannot be removed by simply applying a difference to the log-prices, which often occurs in financial time series. In our case, volatility modelling applies after having removed the structure in the conditional mean as described in the previous section. Once this has been done we can focus on the structure in the conditional variance.

The inspection of Fig. 2 showed the presence of a dependence structure in the conditional variance. This is also confirmed by exploring the autocorrelation functions of the squared residuals as well as by the application of the tests by McLeod and Li [47] or Rodríguez and Ruiz [48]. Fig. 3 shows the ACF of the squared residuals of electricity prices, where some significant coefficients can be observed in the first lags. Similar results (in terms of significance of the first autocorrelation coefficients of the squared residuals) are obtained for the rest of the series considered in this paper. The p-values of the McLeod-Li test for these series are respectively 9.4854e-004, 0 and 1.3071e-006, which indicates the rejection of the null hypothesis about the non-existence of Auto Regressive Conditional Heteroskedasticity effects. Besides, they present excess kurtosis: 4.6509, 11.5403 and 19.9768, respectively.

The basic model that allows for representing uncorrelated series with excess kurtosis and autocorrelated squares is the ARCH(1) model developed by Engle [49], which is given by:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \quad \varepsilon_t \sim NIID(0, 1), \\ \sigma_t^2 &= \omega + \alpha a_{t-1}^2, \end{aligned} \quad (9)$$

where $\omega > 0$ and $\alpha \geq 0$ for σ_t^2 . These two conditions need to hold so as to σ_t^2 being strictly positive and not to degenerate.

The generalization of the ARCH(1) model is the ARCH(p) where the conditional variance σ_t^2 is given in terms of last p lags of the residuals a_t instead of just the first lag as occurs in the ARCH(1) model. The expression of an ARCH(p) is the following:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \quad \varepsilon_t \sim NIID(0, 1), \\ \sigma_t^2 &= \omega + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \dots + \alpha_p a_{t-p}^2. \end{aligned} \quad (10)$$

To avoid the problems in estimation derived from needing a very large order p for an adequate representation of the features of real data, some restrictions can be applied to the coefficients α_j , where $j = 1, \dots, p$. Bollerslev [50] used an approximation for the infinite polynomial of the Wald representation by the ratio of two finite polynomials, usually of very low orders. As a result of this, Bollerslev proposed the GARCH(p, q) model given by:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \quad \varepsilon_t \sim NIID(0, 1), \\ \sigma_t^2 &= \omega + \sum_{j=1}^p \alpha_j a_{t-j}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2. \end{aligned} \quad (11)$$

In practice, the GARCH(1,1) model is usually able to capture adequately the structure in the conditional variance of a great number of real series, not being necessary to increase the orders of the model, p and q .

The parameters of a GARCH model are usually estimated by Maximum Likelihood (ML). The log-likelihood function that is maximized is given by the expression:

$$\log L = -\frac{1}{2} \sum_{t=1}^T \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^T \frac{y_t^2}{\sigma_t^2}. \quad (12)$$

Univariate GARCH(1,1) models could be fitted to each series of residuals, but according to Fig. 2, it would be more interesting to take advantage of the co-movements detected in the volatilities,

using a multivariate model for the conditional variances that allows to model them jointly, so as to understand the dynamic structure of the prices and volatilities of several related energy markets.

The equation of a Multivariate GARCH model (m-GARCH) is given by:

$$R_t = (H_t)^{1/2} \varepsilon_t, \quad (13)$$

where $(H_t)^{1/2}$ would be in our case a 6×6 positive definite matrix such that H_t is the conditional covariance matrix of the m -dimensional ($m = 6$ in our case) vector of residuals obtained after removing the structure in the conditional mean as described in Section 3.

Concerning the second equation needed for the specification of the multivariate GARCH model, the most direct generalization of the GARCH(1,1) model to the multivariate case is the VEC model proposed by Bollerslav et al. [51]. However, the estimation of this kind of models presents several drawbacks. First of all, the number of parameters increases very quickly with the number of series considered. Additionally, since H_t is a covariance matrix, positive definiteness has to be ensured, which introduces more difficulties in the estimation process.

A possible and powerful alternative that allows to consider the multivariate structure of the volatility but reducing the number of parameters to estimate are factor models such as the one developed by Harvey et al. [29]. Dynamic factor models have been extensively used for macroeconomic and financial data, both for capturing the common features in conditional mean and variance, but its use is not so common in applications related to energy markets, although according to recent works Alonso et al. [19,40] they have a great potential in this field, at least in modelling and forecasting series of electricity prices.

Here, we propose a slightly modified version of the model proposed by Harvey et al. [29] to extract common volatility factors from the 6-dimensional series of prices considered in this paper. In the paper by Harvey et al. [29] the structure in the conditional mean of the 7 series of exchange rates considered was removed just taking a difference on the log-series, and a single factor extracted instead of several ones as we do, which implies imposing additional restrictions in the dynamic factor model estimation to avoid the model being undetermined under rotations. This issue is explained in detail in 4.3.

The extraction of common volatility factors is (in this paper) based on the residuals of the VARIMA models estimated in the previous section, where we have discussed the convenience of using univariate ARIMA models or multivariate VARIMA to forecast the prices, depending on the series considered. Univariate models for electricity and CO₂ and electricity prices, except for very short forecasting horizons, one-day-ahead forecasts, in which the VARIMA including the wind power production obtains better results in terms of forecasting errors, and VARIMA models for the computation of forecasts of fossil fuel prices.

Although depending on the specific series considered we focus the forecasting performance is better using the univariate or multivariate models described, the multivariate models allow to capture the cross-correlation structure in the conditional mean, and the estimated residuals from this model will be used in this subsection to extract the unobserved common volatility factors that will allow to estimate the conditional variance of the original series.

Anyway, although we extracted the common volatility factors from the vector of series of estimated residuals of the univariate

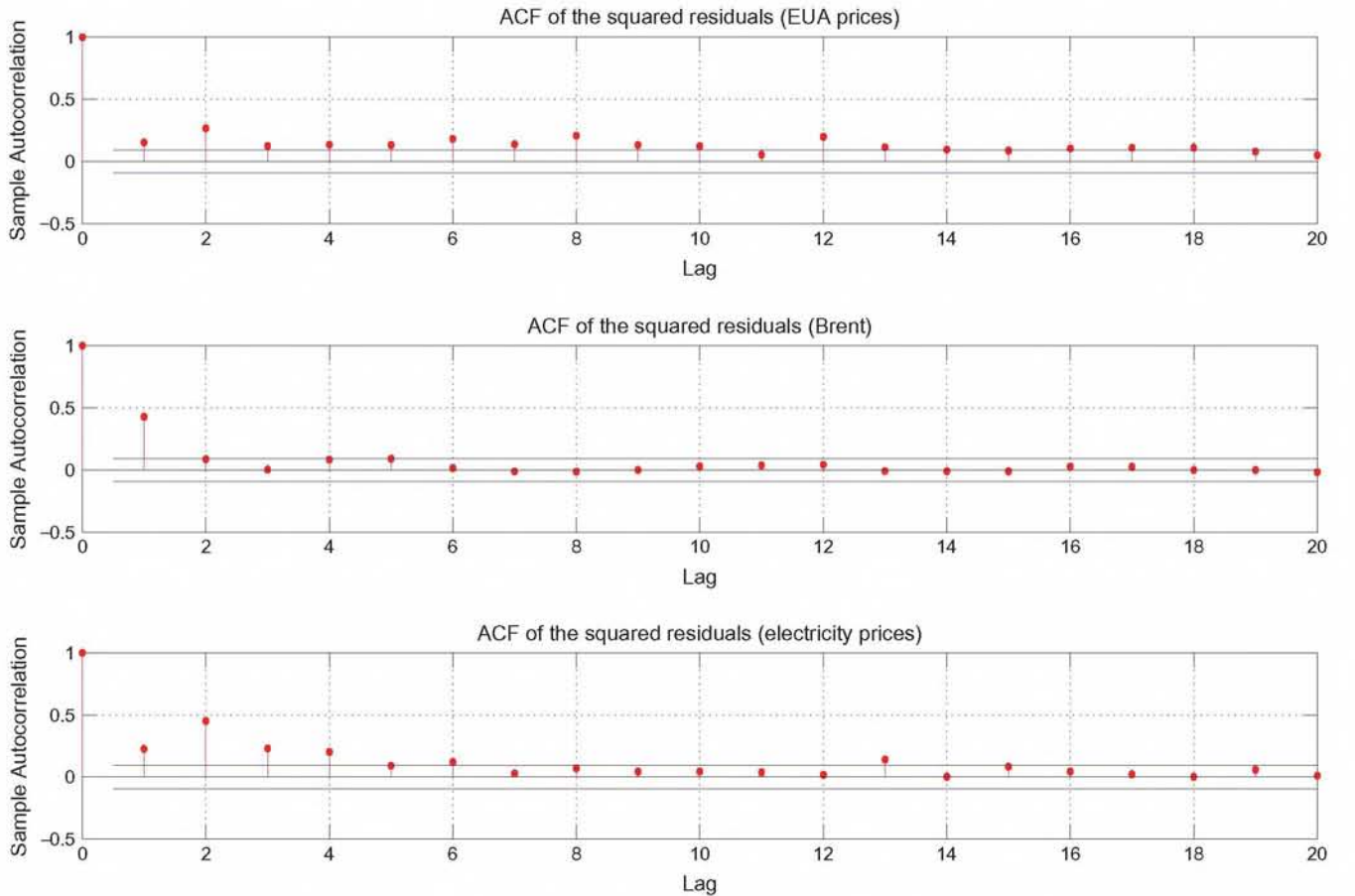


Fig. 3. ACF of the squared residuals of EUAs, brent and electricity prices. Significant coefficients can be observed in the first lags.

models, the results would not differ significantly from these ones, given that here we are accounting for the conditional variance structure and that in the previous section we were focused on the evolution over time of the conditional mean.

A dynamic factor model such as the one presented in this paper allows to extract the common structure in the conditional variance and to use it to estimate the conditional variance of each series.

The equations of the factor model for the volatility would be the following:

$$\mathbf{a}_t = \mathbf{A}\mathbf{F}_t + \mathbf{e}_t, \quad \mathbf{e}_t \sim \text{NIID}(0, \mathbf{S}) \quad (14)$$

$$\mathbf{F}_t = \mathbf{w}_t^*, \quad (15)$$

where \mathbf{a}_t is the m -dimensional vector of series containing the residuals from the VARIMA model previously fitted, \mathbf{A} is a $m \times r$ loading matrix that relates the \mathbf{a}_t with the r -dimensional ($r < m$) vector of conditionally heteroskedastic common factors \mathbf{F}_t . The disturbances \mathbf{e}_t and \mathbf{w}_t^* , which appear in Eqs. (14) and (15) are mutually independent. The variance-covariance matrix of the disturbances \mathbf{e}_t is assumed to be diagonal, so this factors are the so-called specific ones. In the simplest case the disturbances of the transition equation, the conditionally heteroskedastic common factors, follow r univariate ARCH(1) models which according to Eq. (9) are as follows:

$$\begin{aligned} \mathbf{w}_t^* | I_{t-1} &\sim N(\mathbf{0}_{r \times 1}, \mathbf{Q}_t), \quad \text{where } \mathbf{Q}_t = \text{diag}(\sigma_{1t}^2, \sigma_{2t}^2, \dots, \sigma_{rt}^2), \\ \mathbf{w}_{j,t}^* &= \sigma_{jt} \varepsilon_{jt}, \quad \varepsilon_{jt} \sim \text{NIID}(0, 1), \quad \sigma_{jt}^2 = \omega_j + \alpha_j \mathbf{w}_{j,t-1}^{*2}, \\ &\text{for } j = 1, \dots, r, \end{aligned} \quad (16)$$

where I_{t-1} refers to all the available information at time $t-1$. A more general model that could be considered for the common factors is the univariate GARCH(1,1) such that each $\mathbf{w}_{j,t}^*$:

$$\begin{aligned} \mathbf{w}_{j,t}^* &= \sigma_{jt} \varepsilon_{jt}, \quad \varepsilon_{jt} \sim \text{NIID}(0, 1), \quad \sigma_{jt}^2 \\ &= \omega_j + \alpha_j \mathbf{w}_{j,t-1}^{*2} + \beta_j \sigma_{j,t-1}^2, \quad \text{for } j = 1, \dots, r. \end{aligned} \quad (17)$$

The factor model given by Eqs. (14), (15) and (17) allows to model the multivariate structure in the volatility but using a smaller number of parameters than using a multivariate GARCH. These equations can be expressed as an observation and transition equations. This type of models can be estimated under the general framework of state-space models (see [52]) using the Kalman Filter (KF). The log-likelihood function that must be maximized is given by the expression:

$$\log L = -\frac{1}{2} \sum_{t=1}^T \log((2\pi)^n |\Sigma_t|) - \frac{1}{2} \sum_{t=1}^T \mathbf{v}_t \Sigma_t^{-1} \mathbf{v}_t', \quad (18)$$

where both \mathbf{v}_t (innovations) and Σ_t (variance-covariance matrix of the innovations) are direct outputs from the KF.

However, and due to the fact that the disturbances of the transition equation are conditionally heteroskedastic and follow univariate GARCH(1,1) models described by Eq. (17), the standard KF cannot be used, and the augmented version of the Kalman filter, the so called Augmented Kalman Filter (AKF) must be applied.

The practical issues related to the computational implementation of the AKF can be encountered in the paper by Harvey et al. [29] and also in Chapter 6 in the book written by Kim and Nelson [53], where all the details are presented in a more tutorial manner. The key point of the AKF for the conditionally heteroskedastic case is that the disturbances that present conditional heteroskedasticity must play both the role of disturbances and state variable, thus enlarging the dimension of the state vector.

4.3. Application to the extraction of common volatility factors from fuel, CO₂ and electricity prices

Here we illustrate the extraction of common features in volatility (common volatility factors) from the data under study. Based on examination of Fig. 2 as well as on the number of principal components needed to explain a large percentage of the total variability, the number of common volatility factors initially estimated is $r = 3$, assuming that each one follows a GARCH(1,1) model whose parameters need to be estimated.

An additional assumption on the marginal variances of the processes (restriction affecting each set $\omega_j, \alpha_j, \beta_j$ for a fixed $j = 1, \dots, 3$, with $r = 3$ in this case) is needed because the factor model remains unidentified under rotations. It is usually assumed in homoskedastic dynamic factor models [19] that the variance of the noise of the common factors is equal to 1, which in the conditionally heteroskedastic case would imply that $\omega_j + \alpha_j + \beta_j = 1, \forall j = 1, 2, 3$.

Additionally, the elements of the loading matrix \mathbf{A} must also be restricted somehow so that the model is uniquely identified, and Harvey [54] proposed imposing for the elements in \mathbf{A} , i.e. the λ_{ij} , that they are equal to zero when $j > i$.

Here, a GARCH(1,1) specification is assumed for the common volatility factors since this specification is general enough to capture the dependence structure in the conditional variance, but implying a lower number of parameters than in the case of a long ARCH(p) model, i.e., a GARCH(1,1) model is a parsimonious one with a small number of parameters to be estimated.

The estimation is carried out by maximum-likelihood using the AKF as previously mentioned.

Fig. 4 shows the estimation of the conditionally heteroskedastic common factors, $\mathbf{F}_t = (f_{1t}, f_{2t}, f_{3t})'$ as well as the common volatility factors $(\sigma_{1t}^2, \sigma_{2t}^2, \sigma_{3t}^2)'$.

The evolution over time of the conditional variances of the common factors (common volatility factors) shown in Fig. 4 and the patterns of the conditional variances of the series under study depicted in Fig. 2 are clearly related.

A visual inspection of the aforementioned Figs. 4 and 2 shows that the first common volatility factor mainly describes the conditional variance of electricity prices. The evolution over time of the second common volatility factor is related to the volatility of the CO₂ prices (EUAs and CERs) and the third one corresponds to the volatility of the fuel prices (oil, gas and coal). Thus, the common factors, apart from accounting for the multivariate structure of the conditional variance of the data reducing the numbers of parameters to be estimated in comparison with a multivariate GARCH model, have additionally an interesting interpretation in terms of the 3 groups of variables related to energy markets under study (electricity, emission allowances and fossil fuels).

Fig. 5 shows the estimated elements in the loading matrix \mathbf{A} (the loads), whose interpretation is in accordance with those given in the previous paragraph. The first conditionally heteroskedastic common factor is mainly accounting for the volatility of electricity prices. The second one describes the EUAs and CERs (and gives also a much smaller load to oil price). The last one describes the behavior of the volatility associated with fossil fuel prices.

The adequate modelling of the multivariate volatility, which includes taking account of its multivariate dynamic structure when it does exist (as it is the case of the data under study) would firstly allow to properly and accurately estimate the conditional variance of each series, as it is provided in this section. Then, it is worth mentioning that the adequate estimation of the dynamics of the variance-covariance matrix is very important for the computation of forecasting intervals, whose width should not be constant, but wider or narrower in high or low volatility periods, respectively. Besides, the correct estimation of the variance-covariance matrix is a crucial starting point for solving other problems in the current

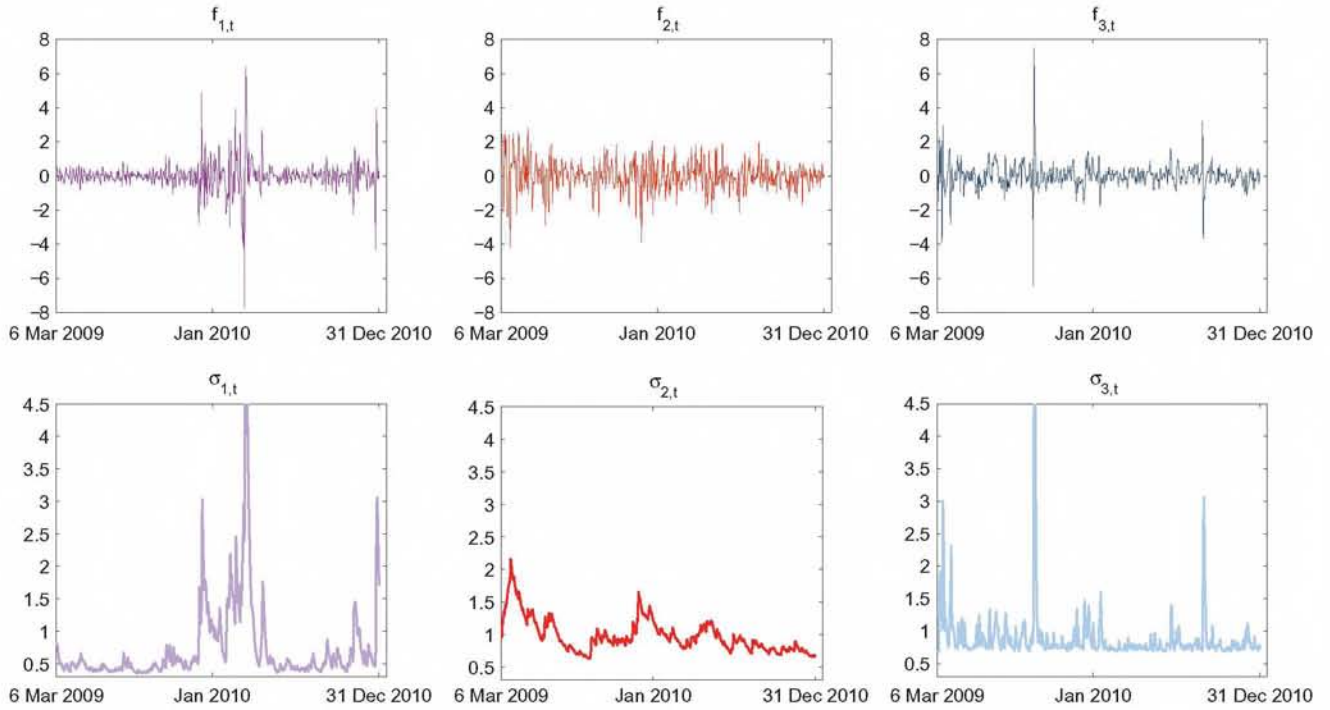


Fig. 4. Conditionally heteroskedastic common factors (first row) and common volatility factors (second row) by plotting the conditional standard deviation extracted from the vector of CO₂, fossil fuels and electricity prices.

context of power markets such as risk management and portfolio optimization. These issues are of interest in large and liquid power

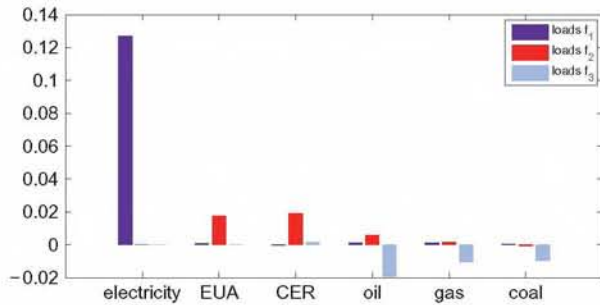


Fig. 5. Loading matrix, relationship between observed series and unobserved common factors.

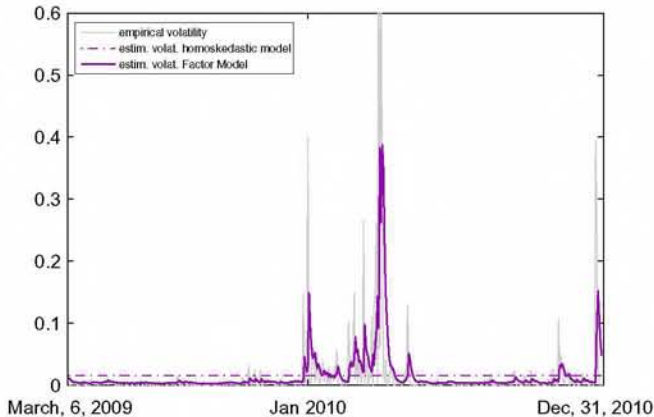


Fig. 6. Conditional volatility of electricity prices (March 2009 to end of December 2010).

markets where the energy derivatives such as forwards and options are well developed, for instance in the European Energy Exchange (EEX) or the ISO-NE (ISO-New England) market.

In Fig. 6 the empirical conditional variance of electricity prices in the period March 2009 till the end of December 2010 as well as the estimation for this conditional variance given by the Factor Model estimated is presented. The estimated variance obtained from the estimation of the homoskedastic VARMA model estimated in Section 3 is also depicted. Estimating the Factor Model for the volatility allows to properly estimate the conditional variance of each series accounting for the multivariate structure that the conditional variances of the considered series present. This will be very important for the accurate computation of volatility forecasts of the series of prices under study, and this will influence the width of forecasting intervals, that will become more accurate (in terms of their coverages).

In Figs. 7 and 8 the evolution over time of the empirical standard deviation, as well as its estimation using the parameter esti-

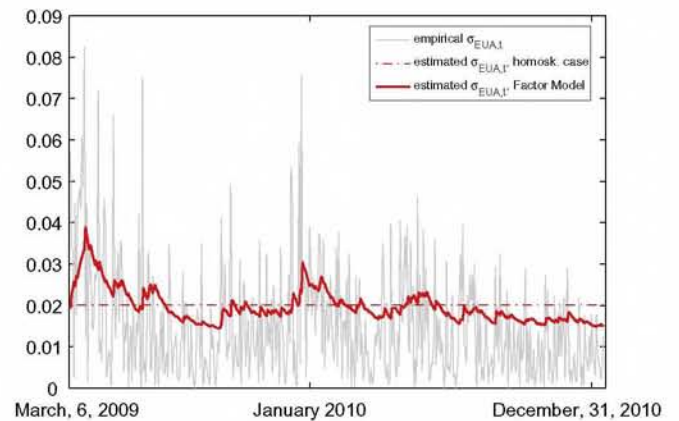


Fig. 7. Conditional volatility of EUA prices (March 2009 to end of December 2010).

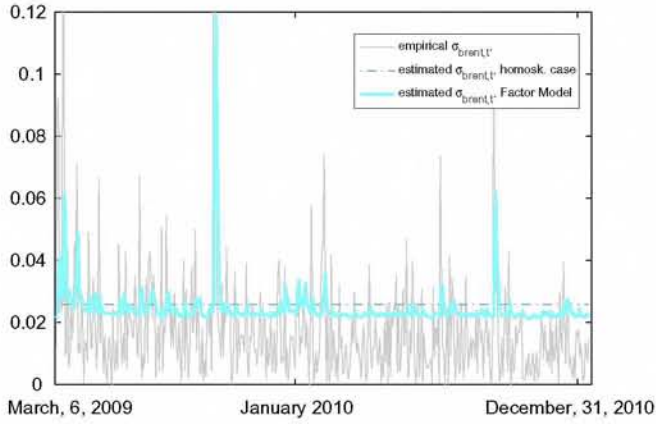


Fig. 8. Conditional volatility of Brent prices (March 2009 to end of December 2010).

mates from the Factor Model for the volatility are shown for the first series from the CO₂ market (EUA price) and for the first series of fossil fuel price (Brent price), respectively. Additionally the constant estimation given by the residual variance of the VARIMA model is jointly depicted.

As it could be expected from Figs. 2 and 4, the differences between the estimates for the conditional variance (or for the conditional standard deviation) of electricity prices and CO₂ prices when assuming or not homoskedasticity are more noticeable than for the Brent price series. Anyway, and even in this case, the Factor Model allows to estimate spikes in volatility that occurred during high volatility clusters. Although for the Brent price series the differences in low volatile periods are not so large, even in these periods there are small differences that will affect the width of forecasting intervals as well as the correct forecasting of the volatility.

The correct estimation of the conditional variance carried out by capturing the dynamic multivariate structure in variance allows to improve volatility forecasts as well as to improve the accuracy of the prediction intervals of the series under study (CO₂, fuels and electricity prices). Investment decisions based on these magnitudes' prices could be improved if the width of the forecasting intervals are adequately computed. Assuming homoskedasticity implies constant width forecasting intervals. However, when the conditional variance evolves over time this could imply an overestimation of the width of the forecasting intervals in low-volatile periods and the computation of narrower intervals than true ones in high-volatility periods.

Finally, the computation of forecasting intervals once the parameters of the model have been estimated can be done by applying the results given by Rodríguez and Ruiz [55], but for the particular case of the model under study as follows:

1. Firstly, a forecast for the volatility of each common factor is computed using the estimates of the parameters of the GARCH(1,1) models for each one: $\hat{\sigma}_{j,T+1|T}^2 = \hat{\omega}_j + \hat{\alpha}_j(\hat{f}_{j,T})^2 + \hat{\beta}_j\hat{\sigma}_{j,T}^2$, $j = 1, 2, 3$.
2. The forecasts for the volatility are used to compute one-step-ahead forecasts of the MSE (Mean Squared Error) matrix, $\mathbf{P}_{T+1|T}$, which in the case of a dynamic factor model where the common factors just present dependence structure in the conditional variance but not in the conditional mean is reduced to $\mathbf{P}_{T+1|T} = \text{diag}(\hat{\sigma}_{1,T+1|T}^2, \hat{\sigma}_{2,T+1|T}^2, \hat{\sigma}_{3,T+1|T}^2)$.
3. Then the prediction MSE is calculated as $\text{MSE}(\tilde{\mathbf{y}}_{T+1|T}) = \hat{\mathbf{A}}\mathbf{P}_{T+1|T}\hat{\mathbf{A}}' + \hat{\mathbf{S}}$, which is also obtained as an output of the KF.
4. Finally the forecasting intervals are computed using the point forecasts included in $\tilde{\mathbf{y}}_{T+1|T}$ and its prediction MSE.

Thus, the capture of the co-movements is relevant for improving forecasting intervals in comparison with those given by univariate conditionally heteroskedastic models that only account for the evolution over time of the conditional variance of each series.

Factor models for the volatility such as the one provided in this Section allow to consider the multivariate structure in the conditional variance avoiding the problems that arise when dealing with multivariate GARCH models and Stochastic Volatility (SV) models, such as the great number of parameters to be estimated as well as ensuring that H_t is a positive definite matrix, which complicates the estimation procedure.

Additionally, the interpretation of the results obtained (common volatility factors estimated when using the Dynamic Factor Model) is very interesting in terms of the different prices under study (electricity, CO₂ prices and fossil fuel ones).

The correct estimation of the volatility is crucial to obtain an adequate evaluation of the risk, and this is very important in financial applications in the energy sector.

5. Conclusions and further research

In this paper we pursue a joint modelling of fuel price, CO₂ emission allowances as well as electricity prices. Although these prices are of great importance for any agent involved in energy markets and some of them are also relevant in macroeconomics, and despite the fact that a lot of attempts have been done to model subsets of them, there are not many published papers that have previously dealt with the multivariate modelling of all of them, both for the conditional mean and variance.

In this paper we follow a VARIMA approach for the prices as well as the univariate modelling of each series, and compared the results in terms of the forecasting error obtained. For the oil, coal and gas prices, the multivariate model improves the prediction accuracy. Additionally, the VARIMA approach gives better results for very short forecasting horizons (one-day-ahead forecasts) in the case of electricity prices. Furthermore, when focusing on one-day-ahead forecasts (forecasting horizon $h = 1$) for electricity prices, incorporating the wind power production in the VARIMA model implies a significant reduction in terms of the out-of-sample prediction. Given the key role that these variables have, not only from the energy sector perspective, but also considering that their influence in the economy of any country is very large, the multivariate modelling is justified for computing more accurate forecasts of some of these series of prices. Additionally, this improvement means that the dynamic cross-correlations between these prices are useful to improve forecasts of some of them.

When focusing on volatility modelling the common features affecting the series of interest are clearly more evident than when focusing on the conditional mean. This allows to exploit it by means of estimating a conditionally heteroskedastic dynamic factor model based on the procedure developed by Harvey et al. [29]. The dynamic factor model here presented allows to account for the multivariate structure in the conditional variance but avoiding the multivariate GARCH models. These models present important drawbacks related to their estimation, due to the need for imposing restrictions, which is not an easy task when dealing with real data, as well as for having to account for positive-definiteness of the conditional covariance matrix.

Furthermore, the estimated common volatility factors and the loading matrix here obtained can be easily interpreted, which gives an additional value to the model considered.

The methodology here described allows to properly estimate the conditional variance of each series accounting for the multivariate structure in the conditional mean. Additionally, we have provided the steps to follow for the computation of volatility forecasts of these

series of prices and how to use them to calculate more precise forecasting intervals whose coverages are closer to the true ones. Although out of the scope of this paper the results obtained and methodologies described can be of interest in financial applications related to the energy sector such as portfolio optimization or risk management for investment decisions under uncertainty.

Other related subject of interest for further research could be the improvement of the multivariate model results by taking into account indexation with respect to other fuels, oil most likely. This could be particularly interesting for the Spanish case, who gets a large fraction of its gas as LNG (Liquified Natural Gas) coming from the spot market, given that usually these prices are not indexed to alternative fuels.

In this paper we studied jointly the evolution over time of several indicators of interest in the current context of power markets, and for this purpose daily prices have been selected. Daily prices or even monthly ones could be in some cases very useful for financial applications, for instance when applying the results obtained in this paper for the correct risk management of the utilities (given that the evolution over time of the variance-covariance matrix has been properly modelled). In these cases the forecasting horizon could be 12, 24 or 36 months and the computation of daily or even monthly forecasts could be enough. However, possible alternatives for dealing with hourly electricity prices and daily prices for all the other magnitudes would be using time series techniques for mixed frequencies extending the methodology developed by Alonso et al. [19] for forecasting hourly electricity prices by incorporating the forecasted wind power production, fossil fuel or CO₂ prices as explanatory variables.

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